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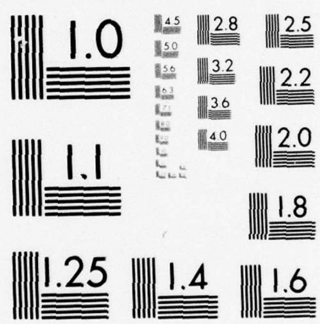
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AN ANALYSIS OF THE LMS ADAPTIVE FILTER USED AS A SPECTRAL LINE ENHANCER

by

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Fleet Engineering Department

February 1975

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ADMINISTRATIVE INFORMATION

This report explains the theoretical basis of the unique properties of the adaptive line enhancer and presents computer simulations of the broadband noise reduction that can be achieved by the device.

Initial funding for the study reported herein was provided under the NUC IRIED program under the direction of Dr. E. Cooper. Additional funding for the completion of this project was sponsored by Mr. J. Neely, NAVSEA 06H1 under the Adaptive Noise Cancelling and Line Enhancing (APACHE) project.

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The adaptive line enhancer is a new device with unique capabilities. The device has significant applications in the enhancement of narrowband spectral lines in a broadband noise field when there is a poor signal-to-noise ratio at the input and there is insufficient a priori information on which to design appropriate filters. The device automatically

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filters out the components of the signal which are uncorrelated in time and passes the correlated portions. Since the properties of the device are determined solely by the input signal statistics, the properties of the filter automatically adjust to variations in the input signal statistics to obtain the LMS approximation to a Weiner-Hopf filter. The device will thus track slowly varying spectral lines in broadband noise. This paper derives the properties of an N-weight adaptive line enhancer for a single stable input spectral line in both an arbitrary and a white noise background. The fundamental properties of the adaptive line enhancer are discussed in terms of these solutions.

SUMMARY

PROBLEM

The adaptive line enhancer (ALE) is a new device with unique capabilities. The device has significant applications in the enhancement of narrowband spectral lines in a broadband noise background when there is a poor sign-to-noise ratio and there is insufficient a priori information on which to design appropriate filters. This paper explains the theoretical basis for the unique properties of the device and presents computer simulations of the broadband noise reduction which can be achieved by the device.

RESULTS

An analytic solution is obtained for the LMS weight vector and the signal-to-noise enhancement of an N-weight adaptive line enhancer for a single stable input spectral line in both an arbitrary broadband noise and white noise background. It is shown that the ALE forms a narrowband filter in which the passband is centered on the frequency of the input spectral line. It is shown that, for stable lines, the signal-to-noise enhancement depends only on the number of weights and is independent of the input signal-to-noise ratio. It is shown that the ALE adjusts the phase of the time-delayed input signal so that the components of the two channels are in phase and sets the amplitude of the filter weights so that the total signal and noise power within the filter passband is equal in the two parallel channels.

RECOMMENDATIONS

The results of this analysis for a stable spectral line substantiate the properties of the ALE that have previously been observed experimentally and by computer simulation. It is now of interest to determine to what extent these results can be extended to include the properties of the ALE for input signals consisting of multiple, non-interfering spectral lines and/or time-varying or short-duration input spectral lines. An analytic solution to the time-varying case requires that the magnitude of the misadjustment noise be included in the solutions and is of particular interest in predicting the capabilities and appropriate design

parameters of the ALE in practical applications. Initial results indicate that the time-varying case can be solved by a straightforward extension of the results of this paper.

ACKNOWLEDGMENTS

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CONTENTS

I. INTRODUCTION

The adaptive line enhancer (ALE) was first conceived and implemented by J. McCool and B. Widrow. They discovered that a new device could be constructed by alternating the inputs and outputs of an adaptive LMS linear transversal filter [1, 2]. The capabilities of this device are unique, and its applications to the measurement of narrowband spectral lines in a broadband noise field are significant in cases of poor signal-to-noise ratio when insufficient a priori information is available to design appropriate filters. The ALE automatically filters out the components of the input signal which are uncorrelated in time and passes the correlated portions. When the input waveform consists of a set of non-interacting narrowband signals superimposed on a white noise input signal, the ALE constructs a set of narrowband filters whose passbands are centered about each of the narrowband input signals.

The device thus approximates a set of matched filters in which the filter passbands are determined automatically, solely on the basis of the input signal statistics. No a priori information as to the number of signals, their frequencies, or of the dynamics of their source is required. For stable spectral lines, there is no threshold signal-to-noise ratio required to ensure effective operation of the device and the signal-to-noise enhancement achieved is independent of the input signal-to-noise ratio. Since the ALE is an adaptive filter, the device will automatically adjust the passband of the filter to follow changes in the input signal statistics [2]. The ALE is thus capable of tracking slowly varying spectral lines in broadband noise. The frequency limitations of the device are determined by the input sampling rate, the number of weights, and the weight update rate. The time required to update each weight on current hardware is approximately 500 nsec per weight, and each weight is updated at the input sampling rate.

These properties of the ALE have been verified experimentally and by computer simulations.* The signal-to-noise enhancement of the device is achieved by measuring correlations between the signal and noise components of the input waveform. Since the signal is

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correlated in time and the noise is uncorrelated, the processing gain of the device can be increased for stable lines by increasing the number of filter weights. This paper provides a theoretical basis for the described properties of the adaptive line enhancer and presents representative computer simulations of the broadband noise reduction which can be achieved by the device. Since the ALE is a variation of the LMS adaptive filter, the basic criteria regarding the convergence and stability of the filter are identical to those derived in Reference 2. In this paper we will derive the LMS weight vector and the gain in signal-to-noise ratio for an N-weight ALE for an input signal consisting of a single stable spectral line in an arbitrary noise background. These results are used to derive specific solutions for a white noise input. These solutions illustrate the fundamental properties of the ALE and provide a basis for extended analysis to include the properties of the ALE for input signals which consist of multiple spectral lines and/or time-varying or short-duration input spectral lines.

II. ADAPTIVE LINE ENHANCER

An ALE for real input signals is shown in Figure 1. As incidated in Figure 1, the input signal X(j) is fed directly to the positive port of a summing function and is simultaneously fed through a parallel channel in which it is delayed, passed through an adaptive linear transversal filter and then subtracted from the instantaneous signal. The difference between these two signals is the error signal $\epsilon(j)$. $\epsilon(j)$ is multiplied by a gain μ and fed back

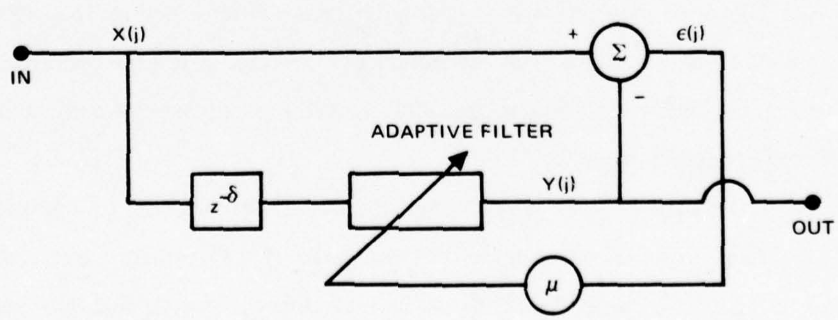

Figure 1. The Adaptive Line Enhancer: X(j) is the input waveform, $z^{-\delta}$ is a fixed time delay; Y(j) is the output of the adaptive filter and the ALE; Σ denotes the summing junction. $\epsilon(j)$ is the error signal; and μ is the feedback gain.

to the adaptive filter to readjust the weights. The weights of the filter are readjusted until $\epsilon(j)$ is minimized according to the recursive algorithm [2].

$$\overrightarrow{W(j+1)} = \overrightarrow{W(j)} - \mu \overrightarrow{\Delta\epsilon}_j^2 \approx \overrightarrow{W(j)} + 2\,\mu\epsilon(j)\,\overrightarrow{X(j)} \tag{1}$$

Thus, when the mean square error is minimized, $\overrightarrow{W_{j+1}} = \overrightarrow{W_j}$, and the filter is stabilized. This algorithm is obtained by using the method of steepest descent to minimize the mean square error as discussed in Reference 2. It was shown that for steady-state input statistics, as the integration time becomes long, the weight vector that minimizes the mean square error is equivalent to the Wiener-Hopf solution, i.e.,

$$\overrightarrow{W_{LMS}} = \Phi^{-1}(X', X)\,\overrightarrow{\Phi(X', X)}, \tag{2}$$

where $\vec{X'}$ is the sampled, time-delayed signal vector, and $\vec{X}$ is the instantaneous signal at the positive port of the summing function. $\Phi(X', X')$ is the autocorrelation matrix of the sampled, time-delayed signal, and $\overrightarrow{\Phi(X', X)}$ is the crosscorrelation vector between $\vec{X'}$ and $\vec{X}$. It has been shown that Eq. (1) is equivalent to Eq. (2) within the misadjustment noise of the filter [2]. The misadjustment noise is present because of the error that arises in estimating the gradient of the error surface in Eq. (1). The recursive implementation of Eq. (1) thus allows an approximate Wiener-Hopf solution to be obtained in real time and circumvents the difficulty of calculation $\Phi^{-1}(X', X')$ explicitly [2]. The magnitude of the misadjustment noise decreases as μ decreases [2]. A good approximation to Eq. (2) can be obtained by the recursive implementation of Eq. (1) by proper adjustment of the feedback gain parameter. As μ decreases, the time constant of the filter increases and a fundamental limit arises for time-varying spectral lines. The interrelationships between the filter time constant, the feedback gain parameter, and the filter misadjustment noise for time-varying spectral lines in broadband noise will be treated in a subsequent paper. In this paper we determine the solutions achieved by the filter for stable input signals. In this case the filter misadjustment noise can always be made negligible by reducing the value of μ.

To illustrate the essential features of the ALE, we will first discuss the properties of a 3-weight and a 4-weight ALE. These cases can easily be treated analytically since

Φ^{-1} (X′, X′) can readily be derived. For a single complex spectral line in broadband noise, it is possible to obtain an explicit expression for Φ^{-1} (X′, X′) for an N-weight filter by decomposing the matrix into a vector product. It will be shown that the results obtained for real and complex input signals are equivalent and the filter gain and the LMS weight vector will be derived for an N-weight filter for a single spectral line in white noise.

III. PROPERTIES OF 3- AND 4-WEIGHT ALE'S FOR A SINGLE SPECTRAL LINE IN WHITE NOISE

A. PROPERTIES OF A 3-WEIGHT ALE

Consider the 3-weight filter shown in Figure 2 with $\omega_0\delta = \pi/2$ and $\omega_0\Delta = 2\pi/3$. If

$$X(j) = A \sin \omega_0 j + x_{n0}(j) \tag{3}$$

then

$$X'_1(j) = A \cos \omega_0 j + x_{n1}(j) \tag{4}$$

$$X'_2(j) = A \sin (\omega_0 j + 30°) + x_{n2}(j) \tag{5}$$

$$X'_3(j) = A \sin (\omega_0 j - 30°) + x_{n3}(j) \tag{6}$$

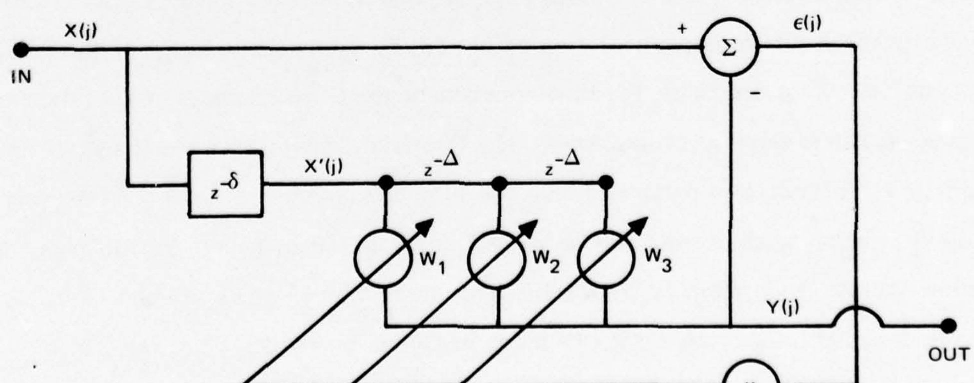

Figure 2. A 3-weight adaptive line enhancer. w_i indicates the value of the weight at the i^{th} tap.

where A is the maximum amplitude of the spectral line and $x_i'(j)$ is the time-delayed signal at the i^{th} tap at the j^{th} input sample time. $x_{ni}(j)$ is the broadband noise component of the input signal at the i^{th} tap, and $x_{n0}(j)$ is the instantaneous noise signal at the j^{th} iteration. If the time delays δ and Δ are sufficient to decorrelate the noise in the two channels, for a white noise field,

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x_{n0}\, x_{ni}\, dt = 0 \tag{7}$$

The input signal correlation matrix is thus given by

$$\Phi(X', X') = \sigma_s^2 \begin{pmatrix} (1 + C) & -1/2 & -1/2 \\ -1/2 & (1 + C) & -1/2 \\ -1/2 & -1/2 & (1 + C) \end{pmatrix}, \tag{8}$$

where

$$\sigma_s^2 = A^2/2 , \tag{9}$$

and

$$C = \sigma_n^2/\sigma_s^2 . \tag{10}$$

σ_s^2 and σ_n^2 are the time-averaged signal and noise power per unit bandwidth, respectively. The crosscorrelation vector between the signals in the two parallel channels is given by

$$\overrightarrow{\Phi(X, X')} = \frac{\sqrt{3}}{2}\sigma_s^2 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}. \tag{11}$$

thus, from Eqs. (2), (8), and (11),

$$\overrightarrow{W_{LMS}} = \frac{\sqrt{3/2}}{(3/2) + C} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \left[(3/2)\,\sigma_s^2 + \sigma_n^2\right]^{-1} \overrightarrow{\Phi(X, X')} . \tag{12}$$

The steady-state output of the ALE is given by

$$Y(j) = \overrightarrow{W}_{LMS} \cdot \overrightarrow{X'(j)} \tag{13}$$

and

$$Y(j) = \left(1 + \frac{2\sigma_n^2}{3\sigma_s^2}\right)^{-1} \left[A \sin \omega_0 j + \frac{1}{\sqrt{3}}\left(x_{n3}(j) - x_{n2}(j)\right)\right] \tag{14}$$

The ALE thus adjusts the weights so that the narrowband signal component of Y(j) is in phase with X(j). The gain in signal-to-noise power is given by

$$G = [(S/N)_{y(j)}/(S/N)_{x(j)}] , \tag{15}$$

where $(S/N)_{y(j)}$ is the ratio of the signal power to the noise power in the output waveform, and $(S/N)_{x(j)}$ *is the signal-to-noise ratio in the input waveform.*

$$(S/N)_{y(j)} = \left[\overline{Y^2(j)}\right]_S \Big/ \left[\overline{Y^2(j)}\right]_N , \tag{16}$$

where $\left[\overline{Y^2(j)}\right]_S$ and $\left[\overline{Y^2(j)}\right]_N$ are the time-averaged signal and noise power components of $\overline{Y^2(j)}$. From Eq. (14),

$$(S/N)_{Y(j)} = \sigma_s^2 \Big/ \frac{1}{3}\left(\sigma_{n3}^2 + \sigma_{n2}^2\right) , \tag{17}$$

For white noise,

$$\overline{\sigma_n^2} = \overline{\sigma_{n0}^2} = \overline{\sigma_{n1}^2} = \overline{\sigma_{n2}^2} = \overline{\sigma_{n3}^2} , \tag{18}$$

and

$$(S/N)_{Y(j)} = \frac{3}{2}\sigma_s^2 \Big/ \sigma_n^2 . \tag{19}$$

Therefore,

$$G = \frac{3}{2} . \tag{20}$$

B. EFFECT ON VARYING INPUT SIGNAL SAMPLING RATE

A variation in the time delay between taps for a given input signal varies the sampling rate and changes the magnitude of the elements of the signal correlation matrix. The gain in the signal-to-noise ratio of the filter thus depends on the details of the sampling process. The filter gain, the LMS weight vector, and the crosscorrelation vector are tabulated for various time delays and sampling rates in Table 1. The dependence of the filter gain on the sampling rate arises because the sampled waveform is not an accurate representation of the input waveform. As the number of weights increases and the sampled waveform becomes an accurate representation of the input waveform, the gain of the filter approaches a value that is determined by the average value of the input waveform and is independent of the details of the sampling process. This will be shown in the next section for an N-weight filter for a single spectral line. (The sampling rate should be asynchronous with the frequency of the input signal to ensure that the sampled waveform is approximately equal to the input waveform.)

Table 1. Properties of 3- and 4-Weight Adaptive Line Enhancers

N	$\omega_0\delta$	$\omega_0\Delta$	G	W_{LMS}	$\Phi(X, X')$
3	90°	120°	3/2	$\frac{\sqrt{3}}{3 + 2C}\begin{pmatrix}0\\-1\\0\end{pmatrix}$	$\frac{\sqrt{3}}{2}\sigma_s^2\begin{pmatrix}0\\-1\\1\end{pmatrix}$
3	60°	60°	2	$(2 + C)^{-1}\begin{pmatrix}1/2\\-\sqrt{3}/2\\1\end{pmatrix}$	$\sigma_s^2\begin{pmatrix}1/2\\-\sqrt{3}/2\\1\end{pmatrix}$
3	90°	90°	1	$(1 + C)^{-1}\begin{pmatrix}0\\-1\\0\end{pmatrix}$	$\sigma_s^2\begin{pmatrix}0\\-1\\0\end{pmatrix}$
4	90°	90°	2	$(2 + C)^{-1}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$	$\sigma_s^2\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$

Although the gain is different in each of the examples in Table 1, note that in each case

$$\overrightarrow{W_{LMS}} = \left[G\,\sigma_s^2 + \sigma_n^2\right]^{-1}\overrightarrow{\Phi(x, x')}\,.$$

Thus if the input signal is given by Eq. (3), and Eq. (7) is satisfied, the crosscorrelation vector is given by

$$\Phi(X, X') = \sigma_s^2 \begin{pmatrix} \cos \omega_0 \delta \\ \cos \omega_0(\delta + \Delta) \\ \cdot \\ \cdot \\ \cdot \\ \cos \omega_0[\delta + (N-1)\Delta] \end{pmatrix} \tag{22}$$

IV. PROPERTIES OF AN N-WEIGHT ALE FOR A SINGLE SPECTRAL LINE IN BROADBAND NOISE

If the input signal is a complex waveform consisting of a single narrowband component, it is possible to decompose the autocorrelation matrix into a vector product and derive the filter gain and the LMS weight vector for an N-weight filter. If there is no correlation between the narrowband signal component and the noise component of the input waveform, the autocorrelation matrix can be separated into two matrices, one expressing the noise correlations between taps and the second expressing the narrowband signal correlations. Thus for an N-weight filter,

$$\Phi(X', X') = P(X_s', X_s') + Q(X_n', X_n') . \tag{23}$$

where P is the narrowband signal correlation matrix, and Q is the noise correlation matrix. For an N-weight filter,

$$P(X_s', X_s') = \begin{bmatrix} p_{11} \; p_{12} \cdots p_{1n} \\ \cdot \\ \cdot \\ \cdot \\ p_{n1} \; p_{n2} \cdots p_{nn} \end{bmatrix} , \tag{24}$$

where

$$p_{ik} = \frac{1}{\sigma_s^2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_{si}'^{*}(j)\, x_{sk}'(j)\, dt. \tag{25}$$

Likewise,

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{lk} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ q_{n1} & q_{n2} & \cdots & q_{nk} \end{bmatrix} \tag{26}$$

where

$$q_{ik} = \frac{1}{\sigma_s^2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x'^{*}_{ni}(j)\, x'_{nk}(j)\, dt\,. \tag{27}$$

For a single complex spectral line,

$$[x'_s(j)]_i = A \exp\,[j\,\omega_0(t + \delta + i\Delta)]\;, \tag{28}$$

where

$$i = 0, 1, \ldots, (n-1)\,.$$

Therefore,

$$p_{ik} = \exp\,[j\,\omega_0\,\Delta(i-k)]\;. \tag{29}$$

For a single complex spectral line, it is thus possible to define a delay vector, D, and decompose the P matrix into a vector product. From Eq. (29), if

$$D = e^{j\omega\delta} \begin{pmatrix} 1 \\ e^{j\omega_0\Delta} \\ e^{2j\omega_0\Delta} \\ \cdot \\ \cdot \\ \cdot \\ e^{(n-1)j\omega_0\Delta} \end{pmatrix} \tag{30}$$

$$P = DD^{*T}\,. \tag{31}$$

Eq. (2) thus reduces to

$$W_{LMS} = \left(\sigma_n^2 Q + \sigma_s^2 D D^{*T}\right)^{-1} \sigma_s^2 D. \qquad (32)$$

For an arbitrary square matrix A and arbitrary column vectors U and V, it can be shown that [3]

$$(A + UV^T)^{-1} = A^{-1} - \left\{(A^{-1} U)(V^T A^{-1}) \Big/ (1 + V^T A^{-1} U)\right\} . \qquad (33)$$

Using this identity, Eq. (32) reduced to

$$W_{LMS} = \left(\sigma_s^2 Q^{-1} D \Big/ \sigma_n^2\right) \left\{1 - \left(\sigma_s^2 \Big/ \sigma_n^2\right) \left\{D^{*T} Q^{-1} D \Big/ \left[1 + \left(\sigma_s^2 D^{*T} Q^{-1} D \Big/ \sigma_n^2\right)\right]\right\}\right\}. \qquad (34)$$

Equation (34) can be reexpressed in terms of the filter gain. When the signal and noise waveforms are uncorrelated, it can readily be shown that the gain of the filter is identical to that previously derived for an N-element detector array. Eq. (15) thus reduces to [3]

$$G = \overrightarrow{W_{LMS}^{*T}}\, P\, \overrightarrow{W_{LMS}} \quad \overrightarrow{W_{LMS}^{*T}}\, Q\, \overrightarrow{W_{LMS}} \,. \qquad (35)$$

As indicated by Eq. (1), a stable solution for the weight vector of the ALE is obtained by minimizing the output power at the summing junction. It has been shown that for a Wiener processor analogous to the ALE, the minimization of the output power is equivalent to maximizing the gain of the array [3]. By differentiating Eq. (35) and solving the corresponding eigenvalue equation, it has been shown [3] that

$$G = D^{*T} Q^{-1} D \,. \qquad (36)$$

Eq. (36) therefore reduces to

$$\overrightarrow{W_{LMS}} = W_0\, Q^{-1} \overrightarrow{D} \qquad (37)$$

where

$$W_0 = \sigma_s^2 \Big/ (G\sigma_s^2 + \sigma_n^2) . \tag{38}$$

Equations (36) and (37) give the LMS weight vector and the filter gain for an arbitrary broadband noise background. For white noise,

$$Q^{-1} = I , \tag{39}$$

where I is the arbitrary matrix. Thus Eq. (37) reduces to

$$G = N^* = N/2 . \tag{40}$$

N^* is the number of complex weights and N is the total number of weights. (Two linear delay lines are required to process the complex input signal expressed by Eq. (28) since the real and imaginary components must be processed independently. A complex weight thus consists of two taps – one on each delay line [4].) Thus for a white noise input,

$$\overrightarrow{W_{LMS}} = W_0 \overrightarrow{D}. \tag{41}$$

For a real input signal

$$X(j) = A \cos \omega_0 j + x_n(j) , \tag{42}$$

and

$$P = \mathrm{Re}\,(DD^{*T}) . \tag{43}$$

Thus

$$p_{ik} = \cos [(i-k)\, \omega_0 \Delta] . \tag{44}$$

In this case, the P matrix is no longer separable into a vector product, but if the interference between positive and negative frequencies is negligible (i.e., when $\omega_0 >> 1/N$) W_{LMS} is given by the real part of Eq. (41). Therefore, for a real input signal,

$$\overrightarrow{W_{LMS}} = W_0 \,\mathrm{Re}\,(Q^{-1} \overrightarrow{D}) . \tag{45}$$

For white noise, Eq. (45) reduces to,

$$\overrightarrow{W_{LMS}} = W_0 \begin{pmatrix} \cos \omega_0 \delta \\ \cos \omega_0 (\delta + \Delta) \\ \cdot \\ \cdot \\ \cdot \\ \cos \omega_0 [\delta + (N-1)\Delta] \end{pmatrix} \tag{46}$$

Note that Eq. (21) and Eq. (46) are in agreement. Since the filter gain is a scalar, G cannot be determined from Eq. (36) for real input signals. However, as shown in the Appendix, by evaluating Eq. (35), the filter gain also equals N/2 for a real input signal. The gain of the real and imaginary components of a complex ALE are thus equal.

V. DETERMINATION OF THE FILTER OUTPUT AND THE FREQUENCY RESPONSE

The output of the ALE can be determined in terms of Eqs. (13) and (46). If

$$X(j) = A \cos \omega_0 j + x_n(j) , \tag{47}$$

the filter output is given by

$$Y(j) = W_0 \left\{ \sum_{i=0}^{N-1} A \cos \omega_0(\delta + i\Delta) \cos \omega_0 [j - (\delta + i\Delta)] + \sum_{i=0}^{N-1} \cos \omega_0(\delta + i\Delta)\, x_{ni}(j) \right\} \tag{48}$$

Thus, the narrowband spectral components of Y(j) and X(j) are in phase when

$$\sum_{i=0}^{N-1} \sin \omega_0 (\delta + i\Delta) \cos \omega_0 (\delta + i\Delta) = 0. \tag{49}$$

This identity is satisfied in the four cases listed in table 1 but it not true for arbitrary values of δ and Δ. In the asymptotic limit as N becomes large, the summations in Eq. (48) can be evaluated in terms of the average values of the waveforms. As N becomes large,

$$\sum_{i=0}^{N-1} \cos^2 \omega_0 (\delta + i\Delta) \approx \frac{N}{2T} \int_{-T}^{T} \cos^2 \omega_0 t' dt' = N/2 , \tag{50}$$

and

$$\sum_{i=0}^{N-1} \sin \omega_0 (\delta + i\Delta) \cos \omega_0 (\delta + i\Delta) \approx N/4T \int_{-T}^{T} \sin 2\omega_0 t' dt' = 0 . \tag{51}$$

Therefore,

$$Y(j) \approx W_0 \left\{ (AN/2) \cos \omega_0 j + \sum_{i=1}^{N} \cos \omega_0 (\delta + i\Delta) x_{ni}(j) \right\} . \tag{52}$$

For a white noise input, the frequency spectrum of the output is thus equal to

$$\begin{aligned} Y(\omega) = W_0 \Big\{ & 2\pi NA [\delta (\omega - \omega_0) + \delta (\omega + \omega_0)] \\ & + \overline{k(\omega)} (N\Delta/2) \Big[\operatorname{sinc} (\omega + \omega_0) N\Delta/2 + \operatorname{sinc} (\omega - \omega_0) N\Delta/2 \Big] \Big\} , \end{aligned} \tag{53}$$

where $\overline{k(\omega)}$ is the time-averaged noise power per unit bandwidth. The filter output can be determined by transforming Eq. (53). As $N\Delta$ increases, the ALE becomes a narrowband filter, and the narrowband-filtered noise term is given by [5]

$$n(j) = r_n(j) \cos [\omega_0 j + \phi_n(j)] \tag{54}$$

where

$$r_n^2 = n_c^2 + n_s^2 \text{ and } \phi_n = \tan^{-1} (n_s/n_c) . \tag{55}$$

The terms n_c and n_s are determined from a summation over the filter bandwidth, B. If the filter bandwidth is divided into M bands, where $B = M\Delta f$,

$$n_c(j) = \sum_{m=-M/2}^{M/2} A_m \cos (2\pi m \Delta f t + \theta_m) \tag{56}$$

and

$$n_s(j) = \sum_{m=-M/2}^{M/2} A_m \sin(2\pi m\Delta ft + \theta_m) . \tag{57}$$

Equation (53) shows that the filter formed by the ALE is a narrowband filter centered at ω_0, and the narrowband component of the filter output in Eq. (53) is in phase with the input spectral line. The filtered noise term has a random phase factor as indicated in Eqs. (54) - (57), but represents an additional narrowband signal superimposed on the spectral line, which increases the recognition differential of the spectral line. The presence of the filtered noise term accounts for the noise power term in the amplitude factor W_0 in Eq. (38).

VI. SIMULATION RESULTS

Computer simulations of the ALE are presented in this section to demonstrate its transient behavior. Figure 3a shows the comparison of the input and output signal of a 256-weight ALE after 512 iterations (two filter lengths in time) for an input signal-to-noise ratio of −12 dB. The white noise input spectrum was limited to 3250 Hz with a sinusoidal input signal at 1000 Hz. Figures 3b and 3c give the Fourier transforms of the input and output signals shown in Figure 3a. The value of $\mu = 10^{-7}$ in this case and the adaptation is slow due to the small value of μ. Figure 3d shows the output spectrum after 2048 iterations and indicates the enhancement of the signal achieved by the ALE. Figures 4a and 4b show the input and output spectra of a 128-weight ALE with $\mu = 10^{-5}$ and a signal-to-noise ratio of −20 dB for a sinusoidal signal at 1000 Hz and a white noise bandwidth of 3250 Hz. Figure 4b indicates the ability of the ALE to find the spectral line and filter out the broadband noise. Figure 4b shows that, in addition to the spectral line, noise lines may also appear at the output due to short-term correlations in the noise. These additional lines are transient, however, and a comparison between Figures 4a and 4b clearly indicates the signal-to-noise enhancement attainable with the ALE.

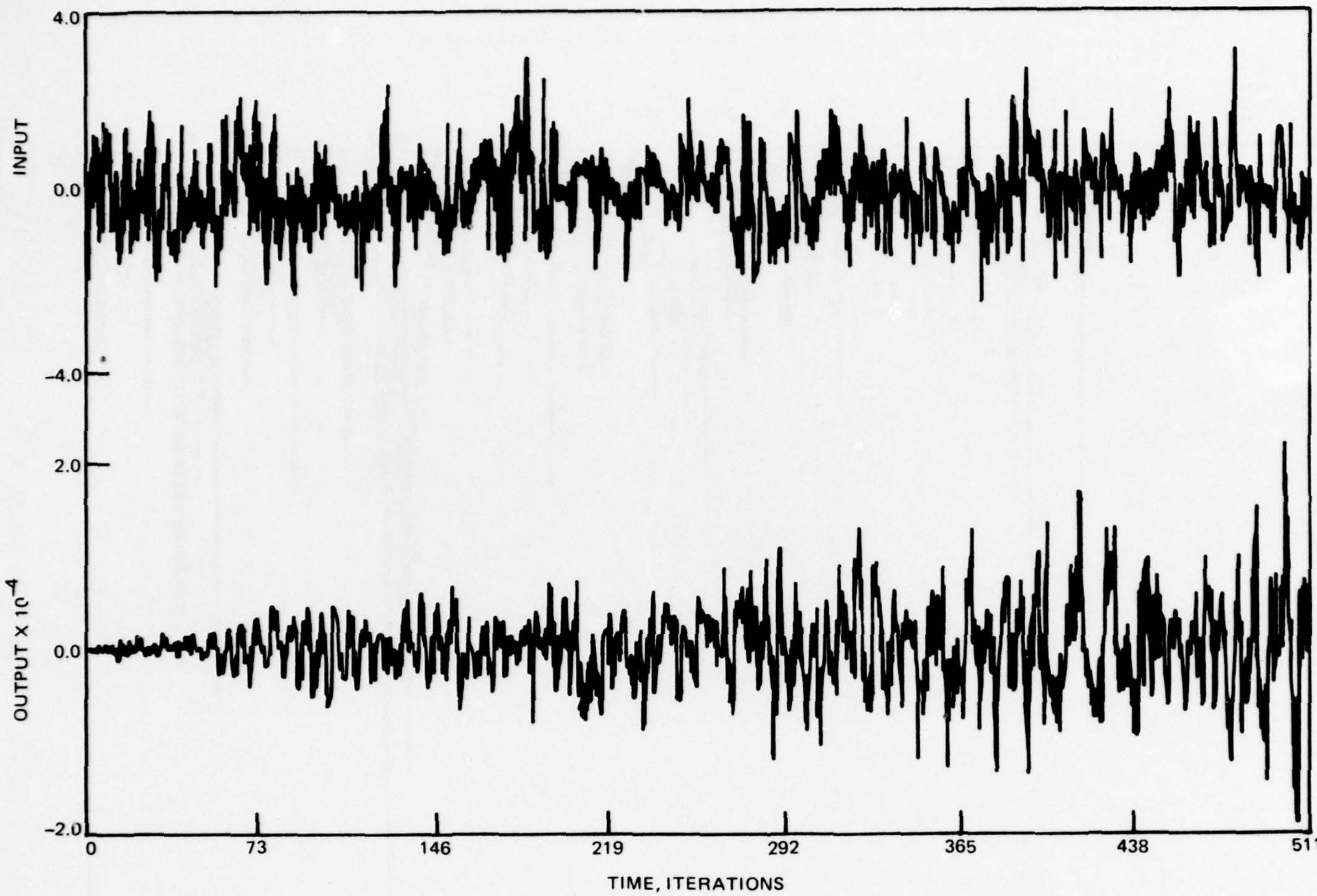

Figure 3a. Input and Output signals from a 256-weight line enhancer with $\mu = 10^{-7}$, S/N = -12 dB, the signal at 1000 Hz, and a noise bandwidth of 3250 Hz. The filter weights are *initially zeroed*. Note that the *magnitude* of the output voltage is significantly less than the input voltage for the first 511 iterations. The output voltage increases in time as the filter adapts, until it reaches the level defined by Eq. (52).

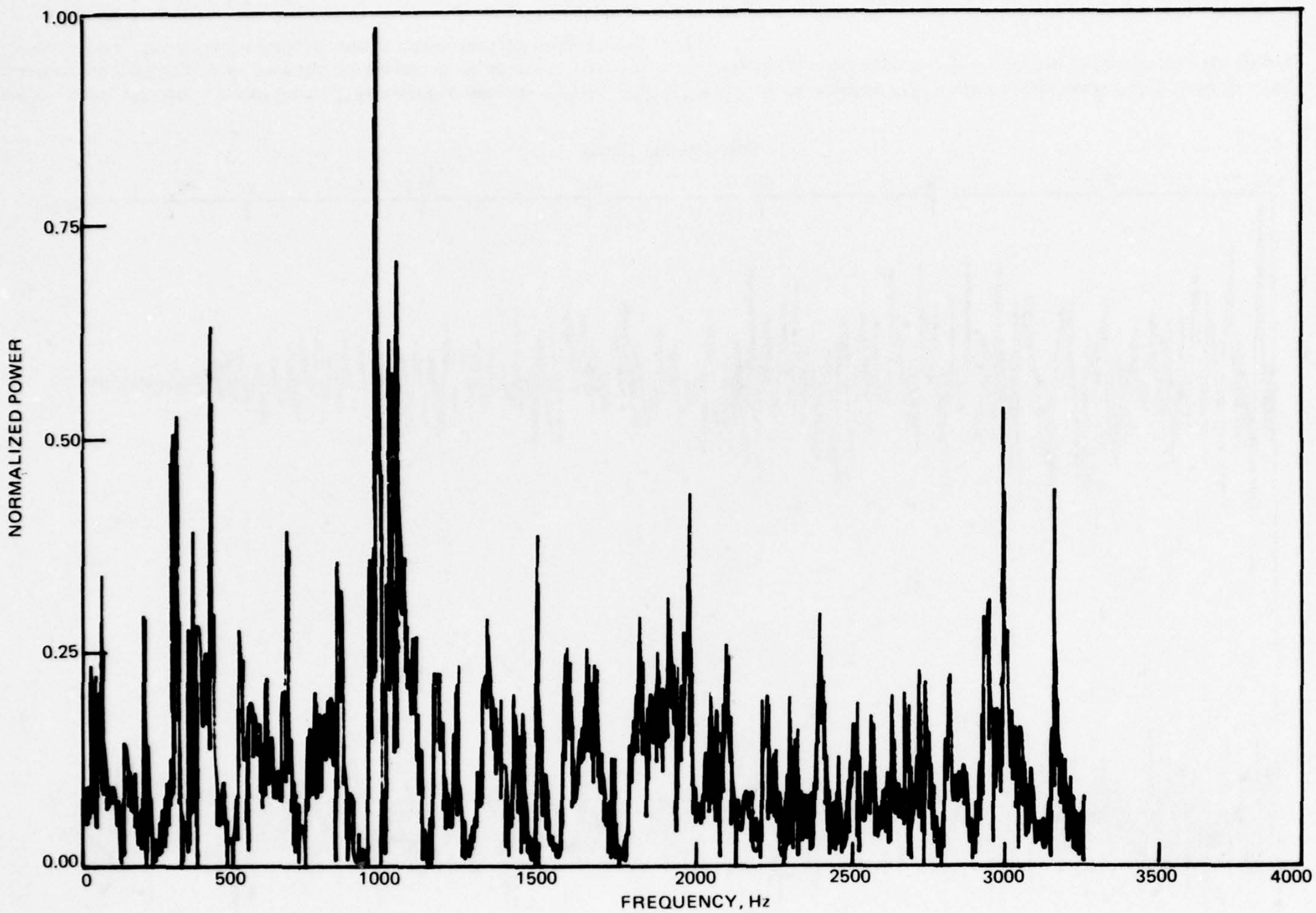

Figure 3b. Input power spectrum of Figure 3a for the first 512 iterations.

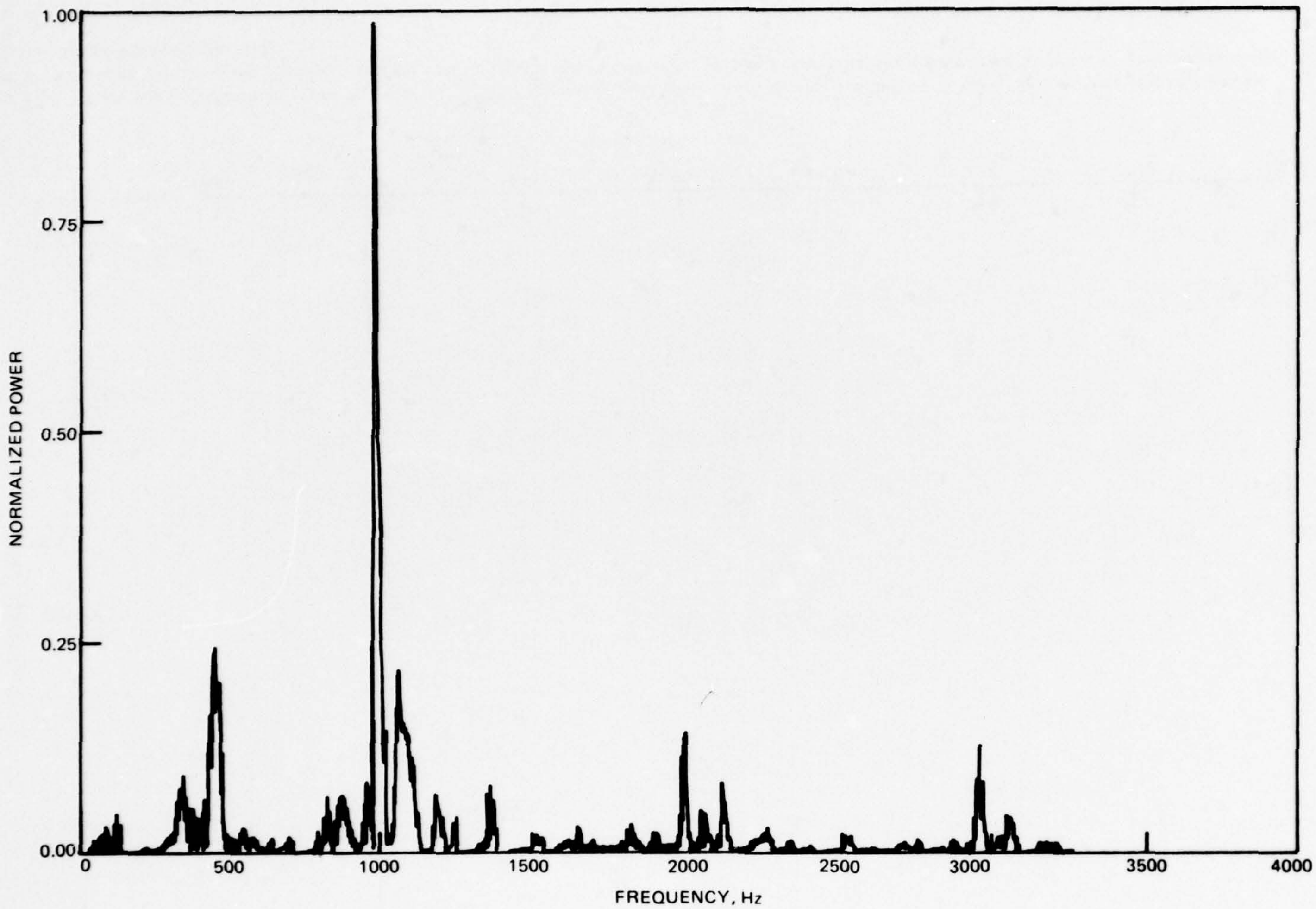

Figure 3c. Output power spectrum of Figure 3a for the first 512 iterations. The noise lines are produced by transients which arise as the filter is adapting. (For a 7-KHz sampling rate, this represents the first 0.07 sec of adaptation.)

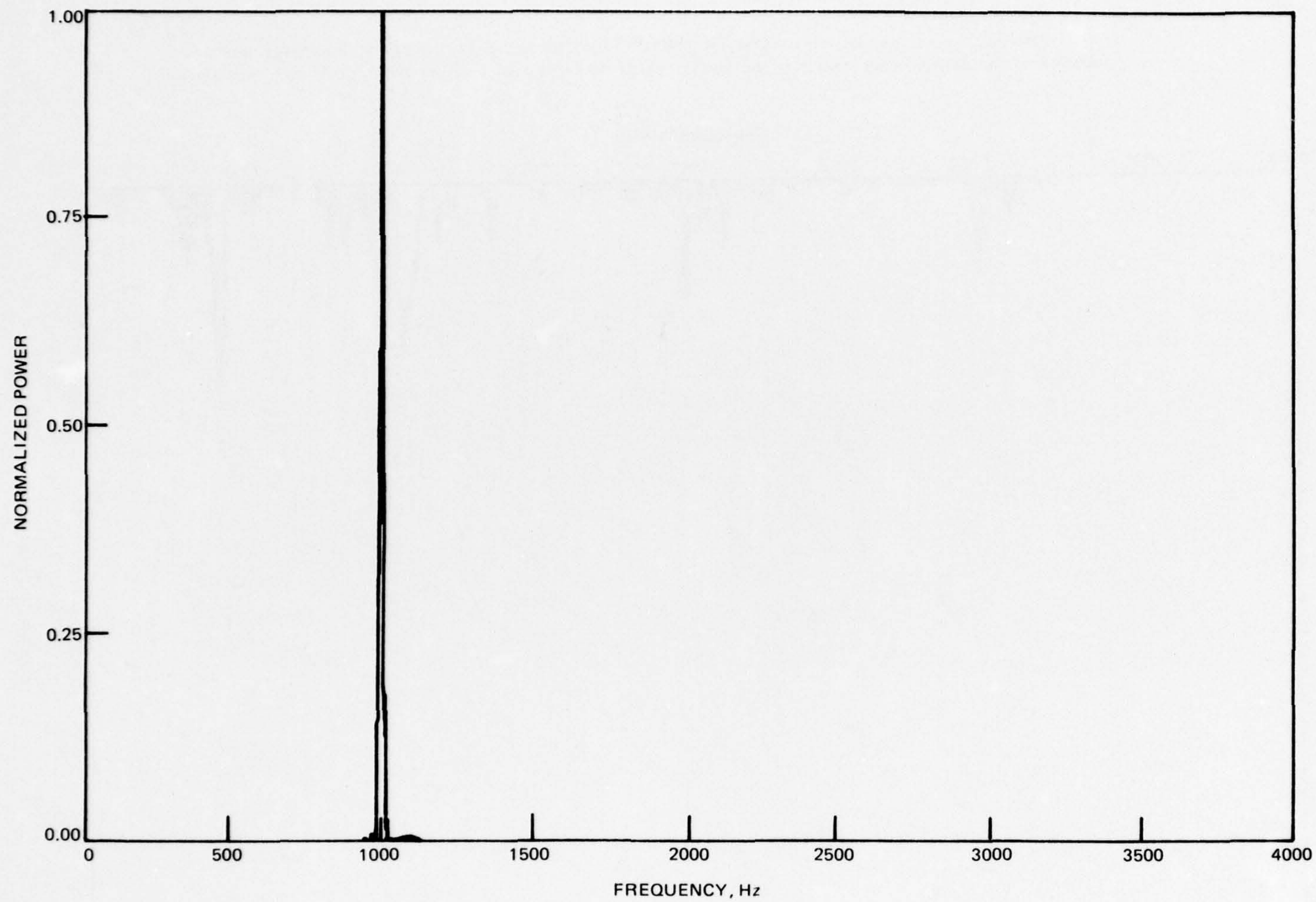

Figure 3d. Output power spectrum of Figure 3a from 1536 to 2048 iterations (for a 7-kHz sampling rate. This represents the 0.22- to 0.29-sec interval from the start of adaptation). Note that the filter has completely adapted to the narrowband signal by this time and that the noise transients present in the spectrum of the first 512 iterations have disappeared.

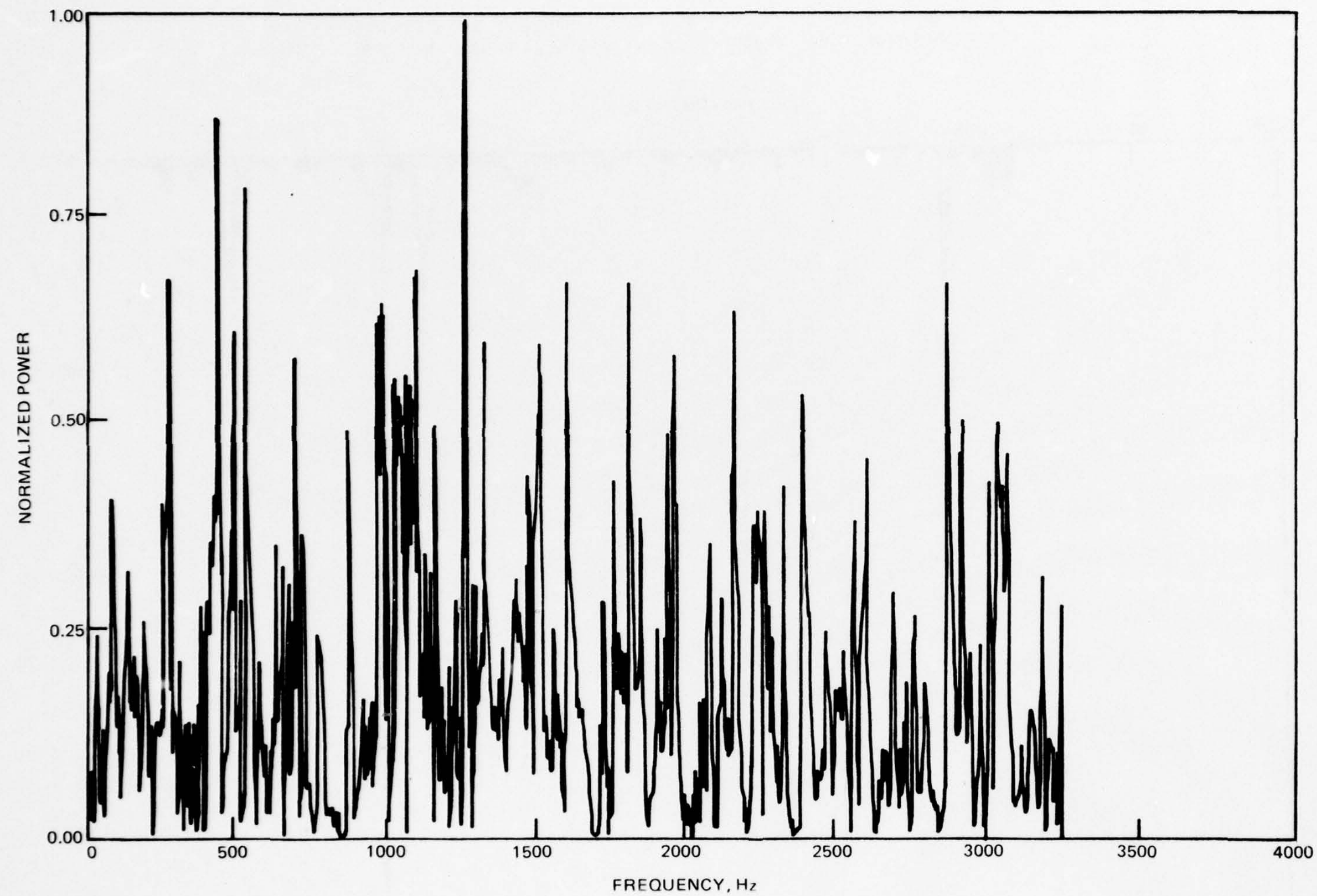

Figure 4a. Input spectrum of a 128-weight ALE with $\mu = 1^{-5}$, S/N = −20 dB, the signal at 1000 Hz, and a noise bandwidth of 3250 Hz.

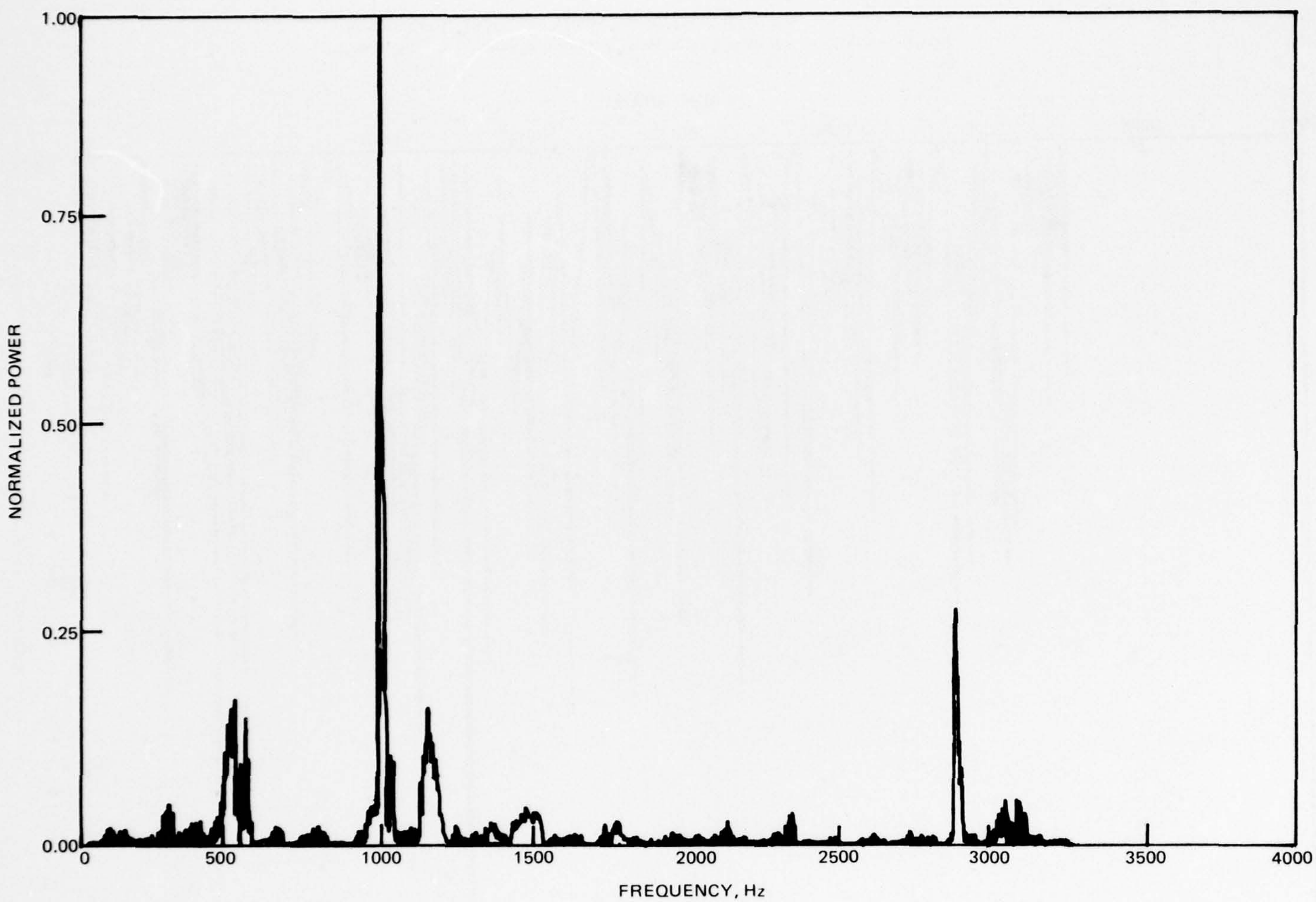

Figure 4b. The output spectrum of Figure 4a after 8704 iterations. (For a 7-kHz sampling rate this corresponds to 1.2 sec from the start of adaptation.

VII. CONCLUSIONS

We have illustrated the procedures by which the properties of the ALE can be derived for a filter with a finite number of weights and derived the asymptotic values of the filter gain and the LMS weight vector when the input waveform consists of a single spectral line in white noise for a filter with a large number of weights (assuming that the sampling time increases as N increases and that the sampling rate is asynchronous with respect to ω_0 so that the sampled waveform equals the input waveform). It is shown that as long as the time delay is sufficient to decorrelate the noise between the two channels of the ALE the basic structure of the LMS weight vector (i.e., the relative magnitude of the individual weights) is independent of the noise power and is determined solely by the spectral components of X(j). The noise power affects $\overrightarrow{W_{LMS}}$ only in terms of the amplitude normalization factor, W_0, as indicated by Eq. (38).

The filter output and frequency response of the filter were derived, and it was shown that the filter adjusts the phase of the time-delayed waveform so that the spectral components of the waveforms in the two channels are in phase at the summing junction. The frequency response of the filter was shown to be proportional to a sinc function centered at the frequency of the input spectral line. Thus, as the number of weights increases (for a fixed sampling rate), the passband of the filter narrows in frequency and the overall gain of the filter increases. As the number of weights becomes large, the filter gain approaches N/2 as indicated by Eq. (40). The amplitude of the weights is adjusted to minimize $\overline{\epsilon^2(j)}$ as indicated by Eq. (1). Equations (52) and (54) indicate that the filter will attempt to adjust the phase and amplitude of the spectral components of the input waveform so that they cancel at the summing junction. For uncorrelated noise, $\overline{\epsilon^2(j)}$ is minimized by filtering out as much of the noise in Y(j) as possible without attenuating the spectral components. The filter output consists of the input spectral line plus filtered noise centered at the frequency of the spectral line. The amplitude normalization factor, W_0, adjusts the gain of the weights until the power of the spectral components at $\omega = \omega_0$ in the two channels are equal. Since the narrowband filtered noise adds to the signal at $\omega = \omega_0$, W_0 is proportional to the filter gain and the input signal and noise power. It would be expected that as the signal-to-noise ratio approaches zero, the output power of the filter would approach zero, since in this case $\overline{\epsilon^2(j)}$

would be minimized by filtering out the entire input waveform. This property is evident since $W_0 \rightarrow 0$ as $\sigma_n^2 / G\sigma_s^2 \rightarrow \infty$. The significant feature of the ALE is that, although $W_0 \rightarrow 0$ as $\sigma_n^2 / G\sigma_s^2 \rightarrow \infty$, the relative magnitude of the individual weights and, consequently the bandwidth of the filter, is independent of the input signal-to-noise ratio. This fact, coupled with the filter gain achieved by performing correlations between the signal and the time-delayed signal, assures effective operation of the devices for stable input signals even when the input signal power is significantly less than the input noise power. It was shown that the ALE does not increase the magnitude of the transmitted spectral lines. Rather, the filter decreases the magnitude of the output power at frequencies outside the passband of the filter. The filter enhances the spectral lines because of the enhanced recognition differential between adjacent frequency bins inside and outside the passband of the filter. The ALE functions as a matched filter that is constructed automatically, with no a priori information, on the basis of the input signal characteristics. Although the values of the gain and the frequency response of the filter derived above are valid only when the input waveform consists of a single spectral line in white noise, it is clear that the principles delineated above also apply in the more general case of non-interfering multiple spectral lines in correlated noise. As shown above, the filter tends to pass the components of the input waveform that are correlated in the two parallel channels of the ALE and to filter out the uncorrelated portions.

The adaptive line enhancer provides a general method of increasing the detection probability of stable spectral lines in broadband noise. The filter parameters were obtained for an arbitrary noise field and specific results were derived for a white noise spectrum at the input. For stable spectral lines, the effectiveness of the device was shown to be determined by the number of filter weights, by the autocorrelation functions of the signal and noise components of the input waveform, and by the crosscorrelation between the signal and noise components of the instantaneous waveform and the time-delayed waveform.

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APPENDIX

DETERMINATION OF THE GAIN OF AN N-WEIGHT ALE FOR THE REAL COMPONENT OF A SINGLE SPECTRAL LINE IN WHITE NOISE

If the input narrowband waveform is expressed by Eq. (42) and the noise background is white, the gain of the filter can be determined from Eq. (35). Therefore,

$$G = \left(\frac{\sigma_n^2 A^2}{\sigma_s^2}\right) \frac{\sum_i \sum_k w_i w_k \frac{1}{2T} \int_{-T}^{T} \cos \omega_0 (t + \delta + i\Delta) \cos \omega_0 (t + \delta + k\Delta)\, dt}{\sum_i \sum_k w_i w_k \frac{1}{2T} \int_{-T}^{T} x'_{ni}(t)\, x_{nk}(t)\, dt} \tag{58}$$

For white noise,

$$\frac{1}{2T} \int_{-T}^{T} x_{ni}(t)\, x_{nk}(t)\, dt = \begin{cases} 0 \text{ for } i \neq k \\ \sigma_n^2 \text{ for } i = k \end{cases} \tag{59}$$

and,

$$G = \left(\sigma_n^2 / \sigma_s^2\right) \left\{ \frac{\sigma_s^2 \sum_{i=1} w_i^2 + \sum_{\substack{i=0 \\ i \neq k}}^{N-1} \sum_{i=0}^{N-1} w_i w_k \frac{1}{2T} \left[\sigma_s^2 (\cos i\Delta \cos k\Delta + \sin i\Delta \sin k\Delta)\right]}{\sigma_n^2 \sum_{i=0}^{N-1} w_i^2} \right\} \tag{60}$$

Using Eq. (46), Eq. (60) reduces to

$$G = 1 + \left\{ \frac{\displaystyle\sum_{\substack{i=0 \\ i\neq k}}^{N-1} \sum_{k=0}^{N-1} \left(\cos^2 i\Delta \cos^2 k\Delta + \frac{1}{4} \sin 2i\Delta \sin 2k\Delta\right)}{\displaystyle\sum_{i=0}^{N-1} \cos^2 i\Delta} \right\} \tag{61}$$

Therefore, if the sampled waveform is an accurate representation of the input waveform,

$$G = 1 + \frac{2}{N} \frac{N^2}{4} - \sum_{k=1}^{N} \left\{ \left(\frac{1}{4} \sin^2 2k\omega_0\Delta + \cos^4 k\omega_0\Delta \right) \right\} \tag{62}$$

Since

$$\frac{1}{2T} \int_{-T}^{T} \cos^4 x \, dx = \frac{3}{8}, \tag{63}$$

Eq. (62) reduced to

$$G = N/2 . \tag{64}$$

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